



# Optimization of thermal resistance of stacked micro-channel using genetic algorithms

Optimization of thermal resistance

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## Abstract

**Purpose** – To determine the optimal dimensions for a stacked micro-channel using the genetic algorithms (GAs) under different flow constraints.

**Design/methodology/approach** – GA is used as an optimization tool for optimizing the thermal resistance of a stacked micro-channel under different flow constraints obtained by using the one dimensional (1D) and two dimensional (2D) finite element methods (FEM) and by thermal resistance network model as well (proposed by earlier researcher). The 2D FEM is used to study the effect of two dimensional heat conduction in the micro-channel material. Some parametric studies are carried out to determine the resulting performance of the stacked micro-channel. Different number of layers of the stacked micro-channel is also investigated to study its effect on the minimum thermal resistance.

**Findings** – The results obtained from the 1D FEM analysis compare well with those obtained from the thermal resistance network model. However, the 2D FEM analysis results in lower thermal resistance and, therefore, the importance of considering the conduction in two dimensions in the micro-channel is highlighted.

**Research limitations/implication** – The analysis is valid for constant properties fluid and for steady-state conditions. The top-most surfaces as well as the side surfaces of the micro-channel are considered adiabatic.

**Practical implications** – The method is very useful for practical design of micro-channel heat-sinks.

**Originality/value** – FEM analyses of stacked micro-channel can be easily implemented in the optimization procedure for obtaining the dimensions of the stacked micro-channel heat-sinks for minimum thermal resistance.

**Keywords** Thermal resistance, Finite element analysis, Optimization techniques

**Paper type** Technical paper

## Nomenclature

$A_b$	= micro-channel base area, $m^2$	$L$	= length of the micro-channel, m
$A_c$	= micro-channel cross-sectional area (= $H_c W_c$ ), $m^2$	$\dot{m}$	= mass flow rate of the fluid, kg/s
$C_p$	= specific heat of fluid at constant pressure, J/kg K	$Nu_{\sqrt{A_c}}$	= nusselt number based on micro-channel cross-section area
$D_h$	= hydraulic diameter of micro-channel, m	$Q$	= heat flux, $W/m^2$
$f_{app}$	= apparent Friction factor	$R$	= thermal resistance per unit area, $^{\circ}C/W/cm^2$
$h$	= heat transfer coefficient, $W/m^2\ ^{\circ}C$	$Re_{\sqrt{A_c}}$	= reynolds number based on micro-channel cross-section area
$H_c$	= height of the micro-channel, m	$T$	= micro-channel wall temperature, $^{\circ}C$
$k$	= thermal conductivity of the micro-channel material, $W/m\ ^{\circ}C$	$T_b$	= maximum temperature of heat sink base, $^{\circ}C$



$T_f$	=	coolant fluid temperature, °C	<i>Greek symbols</i>		
$t$	=	fin half thickness (= $W_f/2$ ), m	$\alpha$	=	aspect ratio = $H_C/W_C$
$t_b$	=	micro-channel base thickness, m	$\mu$	=	fluid viscosity, Ns/m <sup>2</sup>
$v_f$	=	coolant fluid velocity, m/s	$\rho_f$	=	fluid density, kg/m <sup>3</sup>
$W_C$	=	width of a single micro-channel, m			
$W_f$	=	thickness of a micro-channel fin, m			
$W$	=	overall width of a heat sink, m			
$x, z$	=	spatial variable, m			

### Introduction

Heat sinks are designed to dissipate as much heat as possible from the electronic chips. This is crucial to reduce overheating of chips and to increase the chip fatigue life factor. A good heat sink should have minimal thermal resistance in order to dissipate as much heat as possible. The heat sink is defined by some physical parameters such as length, width, channel width, fin width and height of channel. By controlling these physical parameters as well as external parameters such as pumping power, we can obtain the most minimal thermal resistance. In the earlier study of heat sinks, Tuckerman and Pease (1981) designed a micro-channel heat sink, consisting of parallel micro-flow passages of 50  $\mu\text{m}$  wide and 302  $\mu\text{m}$  deep where the thermal resistance obtained was as low as  $9 \times 10^{-6} \text{ K}/(\text{W}/\text{m}^2)$  for a pumping power of 1.84 W.

Following this work, many other research on heat sinks were carried out. Phillips (1990) published a comprehensive review of all micro-channel work to date. Analyses of developing and developed flow, both laminar and turbulent, were presented. Parametric variations of fin to channel width ratio, channel height and aspect ratio, substrate thickness, and channel length were performed. Bar-Cohen and Iyenger (2002) considered various aspects such as minimum material consumption, minimum pumping power along with minimum thermal resistance in developing the air-cooled heat sink system. Wei and Joshi (2004) evaluated the thermal performance of stacked micro-channel heat sinks for a fixed pumping power using a simple resistance network developed by them. They showed that the overall thermal resistance for a two layered micro-channel stack was 30 percent less than for the single layered micro-channel due to doubling of heat transfer area even though the dimensions of micro-channel were not optimized. Later, Wei and Joshi (2003) investigated the effects of number of layers in the stack, pumping power per unit area of heat sink and channel length on the optimal thermal resistance by optimizing the channel configuration.

The objective of the present study is to optimize the stacked micro-channel heat sink to reduce the overall thermal resistance to a minimum, under the constraint of its physical parameters as well as the pumping power for coolant fluid flow. The physical parameters such as fin thickness, channel width, channel length, number of layers and aspect ratio of micro-channel heat sink affect the thermal resistance. Genetic algorithms (GAs) are applied to minimize the thermal resistance with the physical parameters as variables. Three different methods are used to find the overall thermal resistance. First, all the parametric investigation is carried out using thermal resistance model developed by Wei and Joshi (2003). The results obtained are then compared with those obtained by using one dimensional (1D) and two dimensional (2D) finite element methods (FEM). The development of 1D and 2D FEM considered in the present analyses are given in detail in the analysis section.

## Analysis

The general structure of a stacked micro-channel with two layers is shown in Figure 1. A uniform/non-uniform heat flux from the chip may be imposed at the lower surface of the base and the coolant flows inside the channels of each layer. The channels of each layer are separated by fins. A cover plate is bonded to the top of the channel and fin array to confine the coolant.

The overall thermal resistance of the stacked micro-channel is determined by three methods namely; thermal resistance network model, 1D FEM analysis and 2D FEM analysis. Before describing these methods, the correlations for determining the friction factor and the heat transfer coefficient required in all the methods for appropriate flow and heat transfer conditions in the stacked micro-channel under investigation are given.

### *Correlations for friction factor and Nusselt number*

The pressure drop in a stacked micro-channel heat sink includes the contraction and expansion pressure drops in the inlet and outlet, respectively, the pressure drop due to 90° bend, and the pressure drop in the flow direction due to friction called the friction loss. However, the friction loss dominates the pressure drop in laminar flow in a rectangular duct. Therefore, only the friction loss is considered here but its evaluation needs the appropriate value of the friction factor.

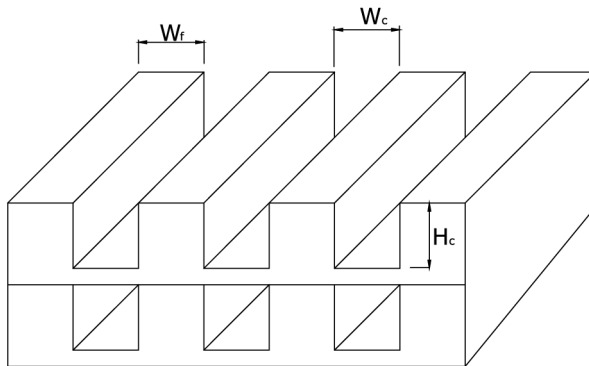
The Churchill-Usagi asymptotic type model as shown in equation (1) is considered for determining the friction factor,  $f_{app}$ .

$$f_{app} \text{Re} \sqrt{A_c} = \left[ \left( \frac{3.44}{\sqrt{y^+}} \right)^2 + 8\sqrt{\pi} G(\alpha) \right]^{0.5} \quad (1)$$

where

$$G(\alpha) = [1.086957^{1-\alpha} (\sqrt{\alpha} - \alpha^{1.5}) + \alpha]^{-1} \quad (2)$$

and



**Figure 1.**  
General structure of a two  
layered stack  
micro-channel heat sink

$$y^+ = \frac{y}{\text{Re}_{\sqrt{A_c}} \sqrt{A_c}} \quad (3)$$

where,  $y$  is the length of the channel and  $\alpha$  is the aspect ratio of the micro-channel defined as:

$$\alpha = W_c/H_c \text{ if } W_c \leq H_c \quad (4)$$

$$\alpha = H_c/W_c \text{ if } W_c > H_c$$

It should be noted that the length scale for Reynolds number is the square root of the channel cross section. Similarly, the Nusselt number for thermally developing condition in a rectangular duct is given as:

$$\text{Nu}_{\sqrt{A_c}}(y^+) = \left[ \left\{ \left( \frac{C1C2 \cdot 8\sqrt{\pi}G(\alpha)}{y^+} \right)^{1/3} \right\}^5 + \left\{ C3 \left( \frac{G(\alpha)}{\alpha^\gamma} \right) \right\}^5 \right]^{0.2} \quad (5)$$

where

$$y^+ = \frac{y}{\text{Re}_{\sqrt{A_c}} \text{Pr} \sqrt{A_c}};$$

$C1 = 1$  for isoflux condition;  $C2 = 0.501$ ;  $C3 = 3.66$  and  $\gamma = 0.1$ .

The above correlation enables the determination of heat transfer coefficient required in the analyses.

### Thermal resistance network model (Wei and Joshi, 2003)

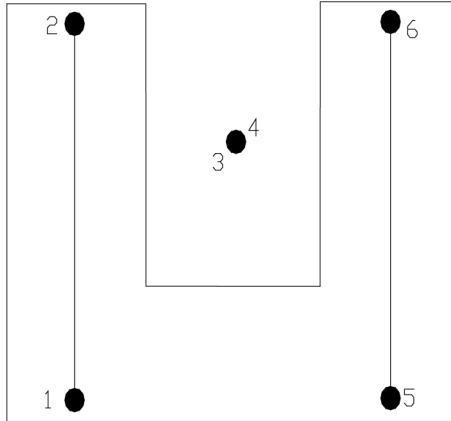
For a micro-channel heat sink, the total thermal resistance comprises of conduction, convection and bulk resistance due to bulk temperature rise of the coolant. Wei and Joshi (2003) have used thermal resistance network to evaluate the total thermal resistance using the above concept, the details of which are available in their paper.

The maximum temperature at the bottom of the base of the first layer,  $T_b$  found iteratively in their scheme, is used to calculate the total thermal resistance,  $R$  as given by:

$$R = \frac{T_b - T_{fi}}{Q} \quad (6)$$

### The finite element model of a stacked micro-channel

Since a stacked micro-channel heat sink consists of an array of repeating channel sections and layers, only one channel of an individual layer is needed to carry out the finite element analysis (Figure 2). The surface above the fins of the top layer of stacked micro-channel is assumed to be adiabatic. In the present study, 1D and 2D FEM is applied to the stacked micro-channel.



**Figure 2.**  
Single element of a  
micro-channel with six  
nodes used for one  
dimensional FEM analysis

### The one dimensional finite element model

In the 1D FEM, the micro-channel can be treated as a combination of two 1D lines in  $z$ -direction with conduction and forced convection, plus a 1D stream of fluid (coolant) passing between the walls. This combination is considered as an element with six nodes as shown in Figure 2. The discretisation of the micro-channel is done by means of subdividing it into several smaller elements. The thickness of the vertical rectangular walls is taken as half of the fin thickness, as shown in Figure 2. The temperature distribution of fluid at a particular cross section is assumed uniform, and varies only along the length of the micro-channel heat sink.

### The governing equations and one dimensional finite element formulation

Assuming the material of the wall to be isotropic, the energy equation for the left wall (conduction in the  $z$ -direction only) in contact with the fluid (forced convection), under the steady-state condition is given by,

$$k \frac{d^2 T}{dz^2} - \frac{h}{t} (T - T_f)_{\text{left}} = 0 \quad (7)$$

Similarly, the energy equation for the right wall is:

$$k \frac{d^2 T}{dz^2} - \frac{h}{t} (T - T_f)_{\text{right}} = 0 \quad (8)$$

The heat transferred from the two vertical walls and the bottom wall to the fluid yields the following equation for the fluid.

$$\dot{m} C_f \frac{dT_f}{dx} = h H_C (T - T_f)_{\text{left}} + h H_C (T - T_f)_{\text{right}} + h W_C (T - T_f)_{\text{bottom}} \quad (9)$$

The governing equation of the bottom wall is not stated explicitly, because the above three equations are adequate to solve the systems of equations involving all the nodes in one element.

*Spatial discretisation for one-dimensional FEM*

The wall is assumed to be a 1D linear element. Therefore, its temperature variation is also linear and is given by:

$$T = [N]\{T\} \quad (10)$$

where

$$[N] = [N_1 \ N_2] = \left[1 - \frac{z}{H} \ \frac{z}{H}\right]; \quad \{T\} = \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} \quad (11)$$

Also, for the fluid, a 1D linear element is considered. Hence its temperature variation is expressed as:

$$T_f = [N]_f\{T\}_f \quad (12)$$

where

$$[N]_f = [N_3 \ N_4] = \left[1 - \frac{x}{L} \ \frac{x}{L}\right]; \quad \{T\}_f = \begin{Bmatrix} T_3 \\ T_4 \end{Bmatrix} \quad (13)$$

The driving potential  $(T - T_f)$  for heat transfer from the corresponding walls of the channel to the coolant fluid in equations (7)-(9) are expressed as:

$$(T - T_f)_{\text{left}} = \frac{1}{2}(T_1 + T_2) - \frac{1}{2}(T_3 + T_4) \quad (14)$$

$$(T - T_f)_{\text{right}} = \frac{1}{2}(T_5 + T_6) - \frac{1}{2}(T_3 + T_4) \quad (15)$$

$$(T - T_f)_{\text{bottom}} = \frac{1}{2}(T_1 + T_5) - \frac{1}{2}(T_3 + T_4) \quad (16)$$

By using Galerkin's method as explained in Segerlind (1984) and Lewis *et al.* (1996), the finite element formulation of equations (7)-(9) are obtained and combined as:

$$[\Lambda_{1D}]\{T\} = \{f_{1-D}\} \quad (17)$$

where, the stiffness matrix  $[\Lambda_{1-D}]$  is given as:

$$[\Lambda_{1D}] = \begin{bmatrix} E_{11} & E_{12} & -e & -e & 0 & 0 \\ E_{21} & E_{22} & -e & -e & 0 & 0 \\ U+V & U & -(U+V+W)-f_m & -(U+V+W)-f_m & V+W & V \\ U+V & U & -(U+V+W)-f_m & -(U+V+W)-f_m & V+W & V \\ 0 & 0 & -e & -e & E_{11} & E_{12} \\ 0 & 0 & -e & -e & E_{21} & E_{22} \end{bmatrix} \quad (18)$$

where

$$E_{11} = e_z + e; \quad E_{12} = -e_z + e,$$

$$E_{21} = -e_z + e; \quad E_{22} = e_z + e,$$

$$e = \frac{hLH_C}{8}; \quad e_z = \frac{tkH_C}{L}$$

$$f_m = \frac{\dot{m}C_f}{2}; \quad \dot{m} = \rho_f W_C H_C v_f;$$

$v_f$  is the velocity of the fluid.

$$U = \frac{-hLH_C}{4}; \quad V = \frac{-hLH_C}{4}; \quad W = \frac{-hW_C H_C}{4}$$

$$\{T\} = [T_1 T_2 \dots T_6]^T \quad (19)$$

and the load vector  $\{f_{1D}\}$  is given as:

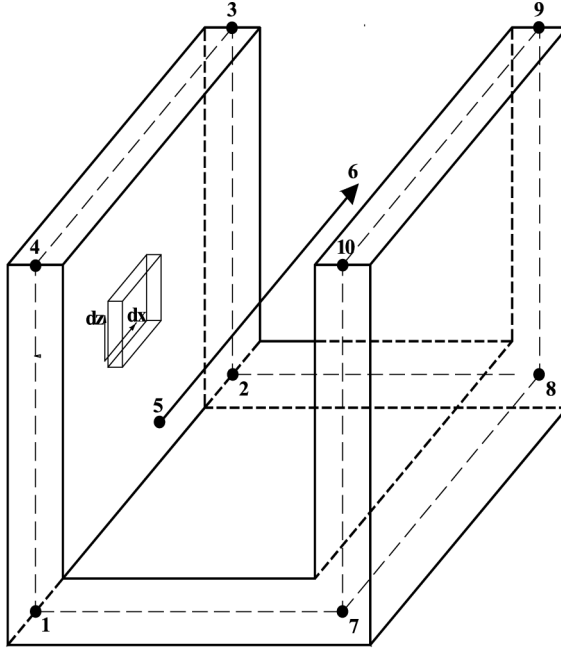
$$\{f_{1D}\} = \frac{Q \cdot A_b}{2} \left\{ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right\} \quad (20)$$

which depends on the distribution of the heat flux on the bottom surface of the micro-channel and the base area,  $A_b$ .

The finite element considered in the 1D FEM analysis has six nodes and, therefore, it will be a  $6 \times 6$  matrix as shown in equation (18) for a single element of the micro-channel.

### The two dimensional finite element model of a stacked micro-channel

For implementing the 2D FEM analysis, one of the stacked micro-channel is treated as a combination of four 2D rectangular walls with conduction and forced convection, plus a 1D stream of fluid (coolant) passing between the walls, except for the topmost layer where the top wall is adiabatic. This combination is considered as an element with ten nodes as shown in Figure 3. The discretisation of the micro-channel is carried out by means of subdividing it into several smaller elements in the coolant flow direction. The thickness of the vertical rectangular walls is taken as half of the thickness of the micro-channel heat sink as shown in Figure 3. The temperature distribution of fluid at a particular cross section is assumed uniform, and is varying only along the length of the micro-channel heat sink.



**Figure 3.**  
Single element of one  
micro-channel with ten  
nodes used for two  
dimensional FEM analysis

### The governing equations and two dimensional finite element formulation

Assuming the material of the wall is isotropic, the energy equation for the left wall (conduction) in contact with the fluid (forced convection), under the steady-state condition is:

$$k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial z^2} - \frac{h}{t} (T - T_f)_{\text{left}} = 0 \quad (21)$$

Similarly, the energy equation for the right wall is:

$$k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial z^2} - \frac{h}{t} (T - T_f)_{\text{right}} = 0 \quad (22)$$

The heat transferred from the two vertical walls and from the bottom wall to the fluid yields the following equation for the fluid.

$$\dot{m} C_f \frac{dT_f}{dx} = hH_C (T - T_f)_{\text{left}} + hH_C (T - T_f)_{\text{right}} + hW_C (T - T_f)_{\text{bottom}} \quad (23)$$

The governing equation of the bottom wall is not stated explicitly because the above three equations are adequate to solve the systems of equations involving all the nodes in one element.



The wall is assumed to be a 2D bilinear rectangular element. Therefore,

$$T = [N]\{T\} \quad (24)$$

where

$$[N] = [N_1 \ N_2 \ N_3 \ N_4] = \left[ 1 - \frac{x}{L} - \frac{z}{H} + \frac{xz}{LH} \ \frac{x}{L} - \frac{xz}{LH} \ \frac{xz}{LH} \ \frac{z}{H} - \frac{xz}{LH} \right];$$

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} \quad (25)$$

However, for the fluid, a 1D linear element is considered. Hence its temperature variation is expressed as:

$$T_f = [N]_f \{T\}_f \quad (26)$$

where

$$[N]_f = [N_5 \ N_6] = \left[ 1 - \frac{x}{L} \ \frac{x}{L} \right]; \quad \{T\}_f = \begin{Bmatrix} T_5 \\ T_6 \end{Bmatrix} \quad (27)$$

The following substitutions are made for the terms in equations (21)-(23).

$$(T - T_f)_{\text{left}} = \frac{1}{4}(T_1 + T_2 + T_3 + T_4) - \frac{1}{2}(T_5 + T_6) \quad (28)$$

$$(T - T_f)_{\text{right}} = \frac{1}{4}(T_7 + T_8 + T_9 + T_{10}) - \frac{1}{2}(T_5 + T_6) \quad (29)$$

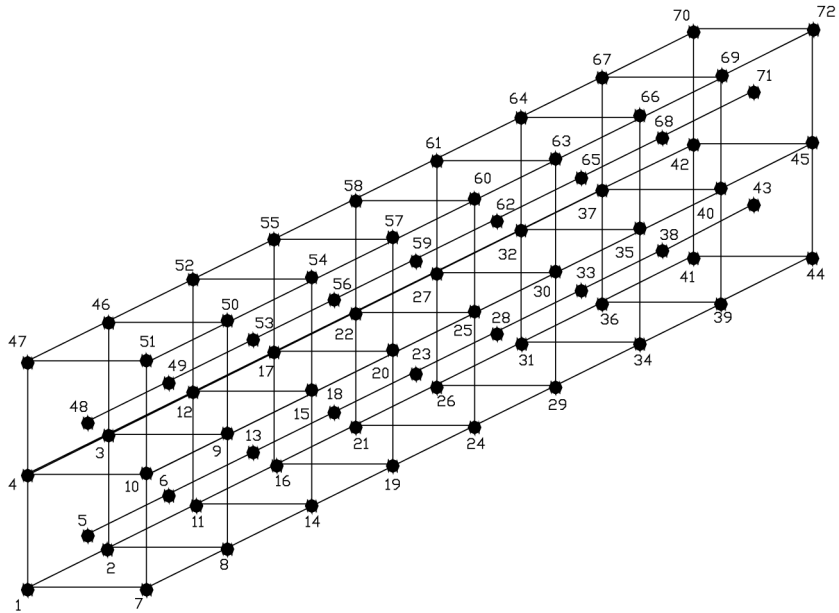
$$(T - T_f)_{\text{bottom}} = \frac{1}{4}(T_1 + T_2 + T_7 + T_8) - \frac{1}{2}(T_5 + T_6) \quad (30)$$

Applying Galerkin's method, similar to equations (7)-(9) for 1D FEM, the finite element formulation of equations (24)-(27) yields:

$$[\Lambda_{2D}]\{T\} = \{f_{2D}\} \quad (31)$$

where  $[\Lambda_{2D}]$  is the stiffness matrix and  $\{f_{2D}\}$  is the load vector for the 2D FEM analysis whose detailed expressions can be found in Quadir *et al.* (2001).

The finite element considered in the present 2D FEM analysis has ten nodes and, therefore, it will be  $10 \times 10$  matrix for a single element of the micro-channel. In Figure 4, the discretisation of a stacked micro-channel with two layers is shown by subdividing a single channel of each layer into eight elements. The different nodes of each element are also numbered in the figure. In all the three methods of analysis considered in the



**Figure 4.**  
Nodal points in each layer  
of eight elements for a  
double stack  
micro-channel

present study, silicon is used for micro-channel fabrication and water is used as coolant with constant properties.

### Genetic algorithms

GAs are based on the Darwinian theory of natural selection and survival of the fittest that exists in the genetics of the species and was first referred to as “genetic algorithms” by Bagley (1967). GAs work by creating a population of individuals where each individual represents a possible solution to a given problem. The fitness function is the most crucial part of the GA. A well formulated fitness function can shorten the optimization time. Then individuals with the best score will be selected via the selection process. The selected individuals are then allowed to reproduce with other selected individuals in the population. The properties of individuals are exchanged through the reproduction processes, namely the crossover and mutation processes. The crossover process is the exchange of respective alleles between chromosomes whereas the mutation process is the change of several alleles in the chromosome. These two processes then produce new individuals, which share some features from the parent. The new individuals will then become a new population of solutions for the next generation. The entire process of evaluation and reproduction then continues until either the population converges to an optimal solution for the problem or the genetic algorithm has run for a specific number of generations.

### Results and discussion

The first objective of the present study is to minimize the overall thermal resistance using GA for different pumping powers per unit area under the constraints as listed in Table I using all the three methods of analysis for a two layer stacked micro-channel.

These constraints considered in the analyses are the same as given by Wei and Joshi (2003). It may be mentioned that the optimized dimensions of the micro-channel obtained by GA are the same as those obtained by Wei and Joshi (2003) where they have used Box Complex optimization method as described in Wei (2000). The study is then carried out to investigate the effect of varying channel length on the optimal thermal resistance for the same two layer stacked micro-channel using all the three methods of analysis. Finally, different number of layers are considered for obtaining optimal thermal resistance for a fixed pumping power of 0.01 W and other required constraints as listed in Table I.

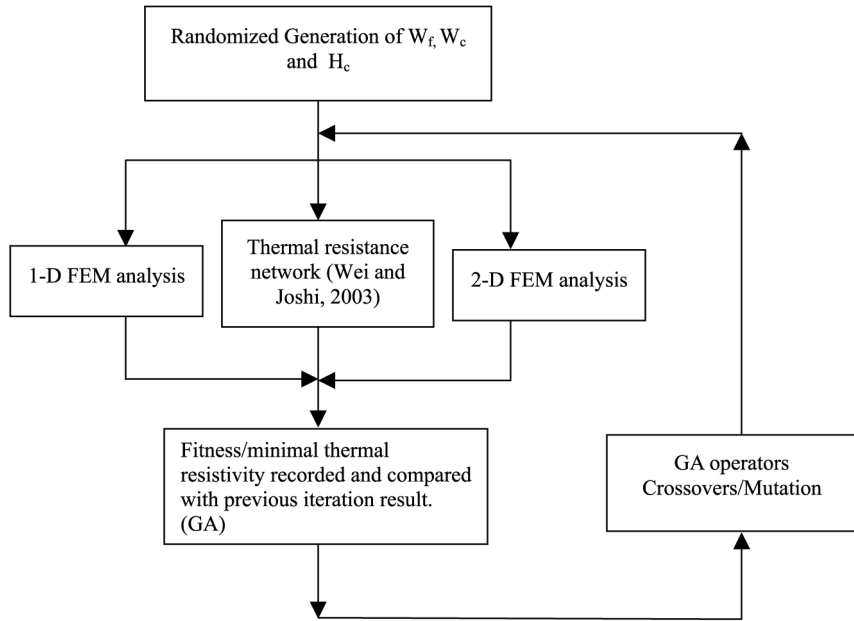
For a fixed channel length, number of layers and pumping power, other physical parameters such as fin thickness, channel width and channel height are made as variables in GA. These variables are then used in the Wei and Joshi's thermal resistance network to determine the thermal resistance. The thermal resistance in the first iteration of GA is stored as the fitness and subsequently the genetic operators, i.e. crossovers and mutations will alter the variables. The altered variables become the new variables for the thermal resistance network in the following iteration of GA. The GA will continue to iterate where the thermal resistance of earlier iteration is compared with the thermal resistance of present iteration. If the thermal resistance of present iteration is less than that of the earlier iteration, then the thermal resistance in the present iteration is stored or otherwise it will be rejected. This process will continue until the thermal resistance converges to a minimal value. This minimal value of thermal resistance as well as the variables become the optimal thermal resistance and optimal physical parameters, respectively. The flowchart for obtaining the above optimal values using GA is shown in Figure 5.

The optimization procedure discussed above is then repeated using 1D and 2D FEM analyses to obtain the corresponding optimal thermal resistance and physical parameters.

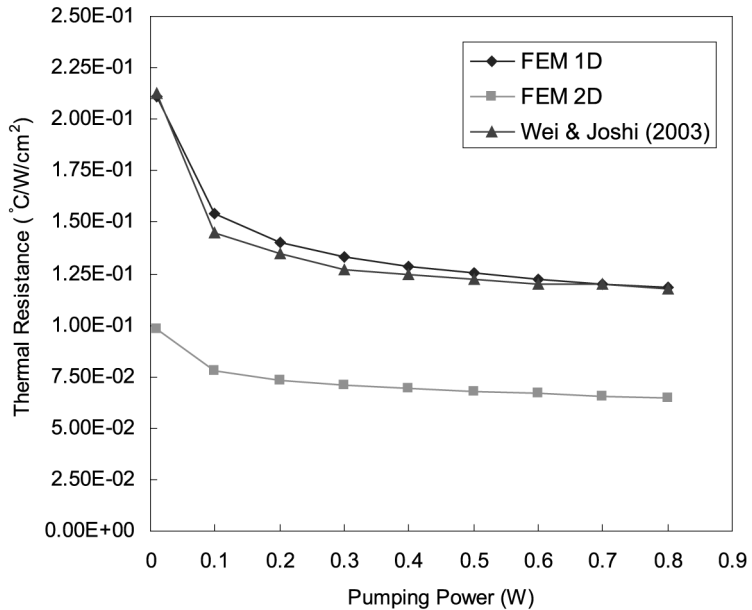
As mentioned earlier, the results of optimization of overall thermal resistance for different pumping powers in a two layer stacked micro-channel using different methods of analysis are shown in Figure 6. The thermal resistance obtained from the resistance network model (Wei and Joshi, 2003) is similar to the 1D FEM analysis. The close proximity of optimal thermal resistance using 1D ( $z$ -direction only) FEM and thermal resistance network suggest the latter approach is a 1D analysis of stacked micro-channel. The thermal resistance obtained by using the 2D FEM analysis is smaller by almost 50 percent compared to those obtained by Wei and Joshi (2003) as well as by 1D FEM analysis. This is because the 2D FEM takes into account the conduction heat flow in  $x$ -direction as well. Thus, it may be concluded that the 2D effect is important in the present analysis. It is also evident from Figure 6 that the thermal

Constraints type	Magnitude
Pumping power density	0.01 W/cm <sup>2</sup>
Pressure drop, $\Delta P$	< 4 bar
Flow rate, $G$	< 1000 ml/min
Length of heat sink, $L$	1 cm
Width of heat sink, $W$	1 cm
Height of a single layer of a heat sink	500 $\mu$ m

**Table I.**  
The constraints used in  
the optimization



**Figure 5.** Optimization flow chart for thermal resistance minimization



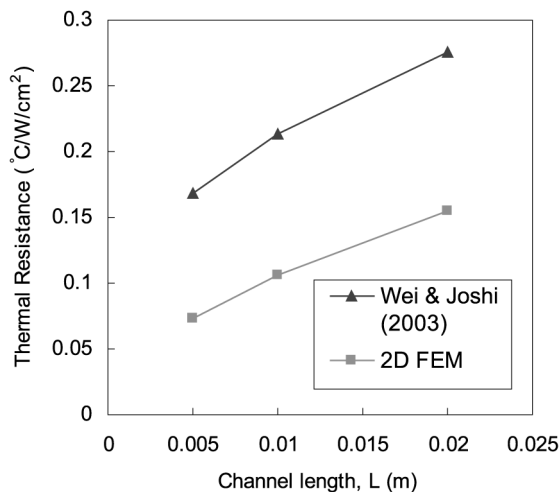
**Figure 6.** The thermal resistance of a double stack micro-channel for different pumping power

resistance beyond the pumping power of 0.3 W does not change significantly for the two layer stacked micro-channel investigated in the present analysis.

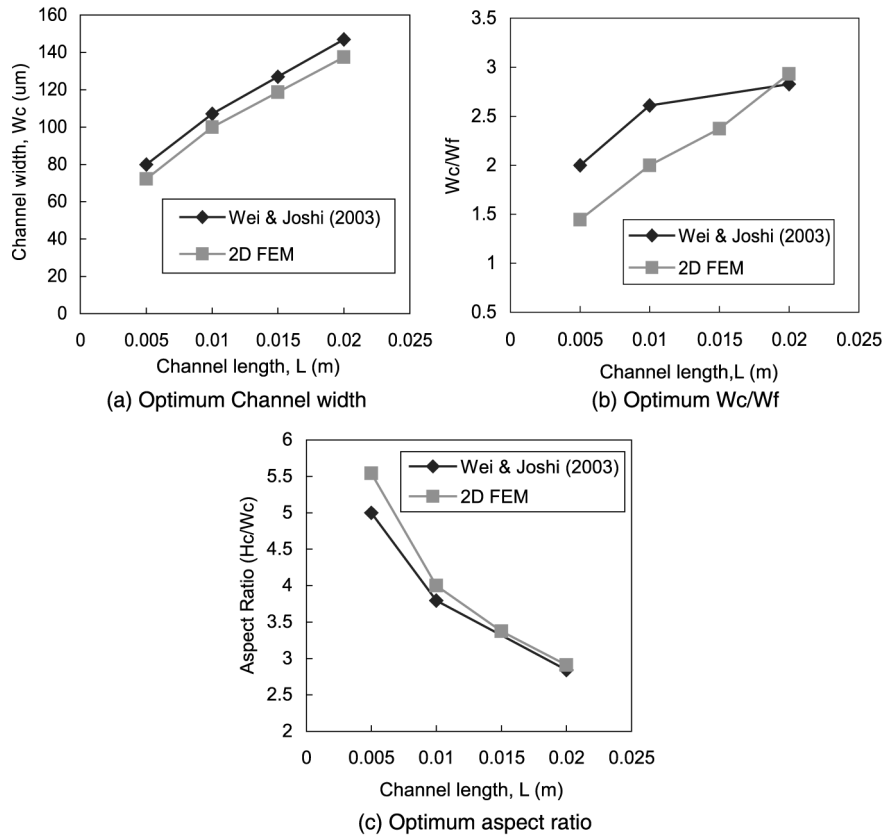
The effect of different channel length on the optimal thermal resistance of a two layer stacked micro-channel is then analysed using the thermal resistance network and 2D FEM analysis and the result is shown in Figure 7 for a fixed pumping power per unit area of  $0.01 \text{ W/cm}^2$ . It can be seen from Figure 7 that as the length increases, the optimal thermal resistance also increases. This trend is predicted by both the methods used in the analysis. The result suggests that short channels instead of long channels should be used to get lower thermal resistances. Further, it can be seen from Figure 7 that the 2D FEM analysis has resulted in lower thermal resistances for all the channel lengths being considered as compared to those obtained by the thermal resistance network analysis. If the surface to be cooled is large, several micro-channel heat sinks with short channels should be integrated, instead of single micro-channel heat sink with long channels (Wei and Joshi, 2003). The above analyses have yielded the optimal channel widths, optimal fin widths and optimal aspect ratios for different channel lengths. These optimal results are shown in Figure 8. It is noticed from Figure 8(a) that the optimal channel width increases with the increase of channel length. This observation is obtained by the resistance network model as well as by the 2D FEM analysis. However, it may be seen that the thermal resistance network model has resulted in larger channel width as compared to that by 2D FEM analysis.

Next, the effect of varying channel length on the optimal ratio of channel width to fin width ( $W_c/W_f$ ) is represented in Figure 8(b). Similar trend of increasing  $W_c/W_f$  with increasing channel length is observed. However, at channel length of 0.02 m, the 2D FEM analysis has given a higher ratio of  $W_c/W_f$  as compared to that by the thermal resistance network model.

The effect of different channel length on the optimal aspect ratio is shown in Figure 8(c). It is seen from this figure that a longer channel requires smaller aspect ratio in order to get the minimum thermal resistance. Both thermal resistance network



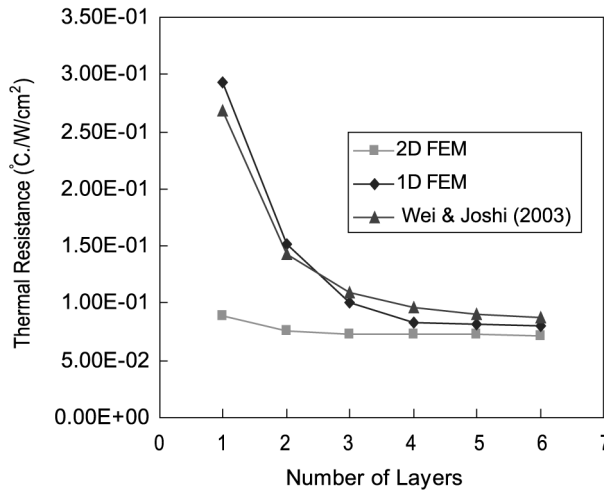
**Figure 7.**  
The optimal thermal  
resistance of a double  
stack micro-channel for  
different channel length



**Figure 8.**  
Effects of channel length,  $L$

model and 2D FEM analysis show close proximity for aspect ratio beyond the channel length of 0.015 m.

Finally, the stacked micro-channel with different number of layers is analyzed using all three methods namely, the resistance network model, 1D and 2D FEM analyses. In each of the method, the thermal resistance is minimized by GA and the optimal thermal resistances, thus obtained, are shown in Figure 9 for different number of layers for a fixed pumping power of  $0.01 \text{ W/cm}^2$ . The results of Wei and Joshi (2003) are close to those obtained by 1D FEM analysis. It can be observed from Figure 9 that the optimal thermal resistances decrease with the increase in the number of layers. This may be explained from the fact that although the heat transfer coefficient is reduced due to lower velocity in the channel due to the increase in layers, the increased surface area results finally in a lower thermal resistance. It is also evident from this figure that the rate of decrease of the thermal resistance is highest for the change in number of layers from one to two. This rate of decreasing slows down considerably when the number of layers exceeds 4 as predicted by the thermal resistance network model and the 1D FEM analysis as well. However, the 2D FEM analysis has resulted in lower thermal resistances for all the number of layers investigated as compared to those given by the thermal resistance network model and the 1D FEM analysis as well. Thus, the 2D heat



**Figure 9.**  
The optimal thermal  
resistance of a double  
stack micro-channel  
having different number  
of layers

conduction effects on thermal resistance in micro-channel with different number of layers is clearly demonstrated. The trend of decreasing the thermal resistance with the increase in number of layers is also seen from the 2D FEM analysis. But it can be observed from this figure that the lowest thermal resistance obtained for five or six layers using the thermal resistance network model or 1D FEM analysis is achieved for two layers only, if the 2D FEM analysis is carried out.

### Conclusions

Thermal resistance of a two layer stacked micro-channel obtained by using the resistance network model, 1D FEM and 2D FEM analyses are minimized by GAs for different pumping power and a fixed channel length. Next some parametric studies are reported for varying channel lengths but for a fixed pumping power. Finally the effect of different number of layers on the thermal performance of the stacked micro-channel for a fixed pumping power of 0.01 W is evaluated. Based on the above analyses, the following conclusions are drawn.

- (1) The thermal resistance reduces with the increase of pumping power for a fixed layer of stacked micro-channel. However, the effectiveness of increasing pumping power decreases for higher pumping powers.
- (2) Wei and Joshi's thermal resistance network model and 1D FEM approach produces almost similar thermal resistances.
- (3) The 2D FEM analysis gave the thermal resistance about 50 percent less than that obtained by the Wei and Joshi's thermal resistance network and 1D FEM. Thus, it is important to take into account the conduction heat flow in the other direction as well.
- (4) The thermal resistance increases as the channel length is increased for a fixed pumping power and a fixed number of layers.
- (5) The optimal number of layers for the minimum thermal resistance for fixed pumping power per unit area of 0.01 W/cm<sup>2</sup> is three based on the 2D FEM approach.

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